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## DEGREASING THE WHEELS OF FINANCE\*

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Can there be too much trading in financial markets? We construct a dynamic general equilibrium model, where agents face idiosyncratic liquidity shocks. A financial market allows agents to adjust their portfolio of liquid and illiquid assets in response to these shocks. The optimal policy is to restrict access to this market because portfolio choices exhibit a pecuniary externality: Agents do not take into account that by holding more of the liquid asset, they not only acquire additional insurance against these liquidity shocks, but also marginally increase the value of the liquid asset, which improves insurance for other market participants.

### 1. INTRODUCTION

Policymakers sometimes propose and implement measures that prevent agents from readjusting their portfolios frequently. Cases in point are holding periods or differential tax treatments, where capital gains taxes depend on the holding period of an asset. Such policies raise a basic question: Can it be optimal to increase frictions in financial markets in order to reduce the frequency of trading? Or, to phrase this question differently: Can the frequency at which agents trade in financial markets be too high from a societal point of view?

The main message of our article is that restricting access to financial markets can be welfare-improving. At first, this result seems to be counterintuitive: How can it be possible that agents are better off in a less flexible environment? The reason for this result is that in our environment the portfolio choices of agents exhibit a pecuniary externality. This externality can be so strong that the optimal policy response is to reduce the frequency at which agents can trade in financial markets; that is, we provide an example of an environment where degreasing the wheels of finance is optimal.

We derive this result in a dynamic general equilibrium model with two nominal assets: a liquid asset and an illiquid asset.<sup>2</sup> By liquid (illiquid), we mean that the asset can be used (cannot be used) as a medium of exchange in goods market trades.<sup>3</sup> Agents face idiosyncratic liquidity shocks, which generate an ex post inefficiency in that some agents have “idle” liquidity holdings, whereas others are liquidity constrained in the goods market. This inefficiency generates an

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<sup>2</sup> Our basic framework is the divisible money model developed in Lagos and Wright (2005). The main departure from this framework is that we add government bonds and a secondary bond market, where agents can trade bonds for money after experiencing an i.i.d. liquidity shock.

<sup>3</sup> We call an asset that can be exchanged for consumption goods *liquid* and one that cannot be exchanged *illiquid*. In our model, the liquid asset is fiat money and the illiquid asset is a one-period government bond. The fact that government bonds cannot be used as a medium of exchange to acquire consumption goods is the consequence of certain assumptions that we impose on our environment as explained in the main discourse of this article.

endogenous role for a financial market, where agents can trade the liquid for the illiquid asset before trading in the goods market. We show that restricting (but not eliminating) access to this market can be welfare-improving.

The basic mechanism generating this result is as follows: The financial market exerts two effects. On the one hand, by reallocating the liquid asset to those agents who have an immediate need for it, it provides insurance against the idiosyncratic liquidity shocks. On the other hand, by insuring agents against the idiosyncratic liquidity shocks, it reduces the demand for the liquid asset *ex ante* and thus decreases its value. This effect can be so strong that it dominates the benefits provided by the financial market in reallocating liquidity.

In a sense made precise in the article, the financial market allows market participants to free-ride on the liquidity holdings of other participants. An agent does not take into account that by holding more of the liquid asset he not only acquires additional insurance against his own idiosyncratic liquidity risks, but he also marginally increases the value of the liquid asset, which improves insurance for other market participants. This pecuniary externality can be corrected by restricting, but not eliminating, access to this financial market.

Our framework is related to the literature that studies the societal benefits of illiquid government bonds, which started with Kocherlakota's (2003) observation that if government money and government bonds are equally liquid, they should trade at par, since the latter constitutes a risk-free nominal claim against future money. In practice, though, government bonds trade at a discount, indicating that they are less liquid than money.<sup>4</sup> Kocherlakota's surprising answer to this observation is that it is socially beneficial that bonds are illiquid. The intuition for this result is that a bond that is as liquid as money is a perfect substitute for money and hence redundant, or in the words of Kocherlakota (2003, p. 184): "If bonds are as liquid as money, then people will only hold money if nominal interest rates are zero. But then the bonds can just be replaced by money: there is no difference between the two instruments at all."

Kocherlakota (2003) derives this result in a model where agents receive a one-time i.i.d. liquidity shock after they choose their initial portfolio of money and illiquid bonds. After experiencing the shock, agents trade money for bonds in a secondary bond market. Many aspects of our environment are similar to Kocherlakota (2003). However, our key result is different and novel. We show that it is not only optimal that bonds are illiquid, but that one needs to go one step further: It can be efficient to restrict the ability of agents to trade them for money in a secondary bond market.<sup>5</sup>

Our article is also related to the macroeconomic literature that studies the implications of pecuniary externalities for welfare (e.g., Caballero and Krishnamurthy, 2003; Lorenzoni, 2008; Bianchi and Mendoza, 2011; Jeanne and Korinek, 2011; Korinek, 2012). In this literature, the fundamental friction is limited commitment; that is, agents have a limited ability to commit to future repayments. Due to this friction, borrowing requires collateral. A pecuniary externality arises, because agents do not take into account how their borrowing decisions affect collateral prices, and through them the borrowing constraints of other agents. As a consequence, the equilibrium is characterized by overborrowing, which is defined as "the difference between the amount of credit that an agent obtains acting atomistically in an environment with a given set of credit frictions, and the amount obtained by a social planner who faces the same frictions but internalizes the general-equilibrium effects of its borrowing decisions" (see Bianchi and Mendoza, 2011, p. 1).<sup>6</sup>

<sup>4</sup> According to Andolfatto (2011, p. 133), the illiquidity of bonds "is commonly explained by the fact that bonds possess physical or legal characteristics that render them less liquid than money ... which raises the question of what purpose it might serve to issue two nominally risk-free assets, with one intentionally handicapped (hence discounted) relative to the other."

<sup>5</sup> Some other papers that study the societal benefits of illiquid bonds are Shi (2008), Andolfatto (2011), and Berentsen and Waller (2011). All these papers show, among other things, that Kocherlakota's result holds in a steady-state equilibrium as well. We present more details of the Shi (2008) framework and compare it to our model and Kocherlakota (2003) in Section 7.

<sup>6</sup> Related to this literature are studies on financial accelerators (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997) or endogenous borrowing constraints (e.g., Kehoe and Levine, 1993; Berentsen et al., 2007).

This pecuniary externality effect has been used to study *credit booms and busts*. In a model with competitive financial contracts and aggregate shocks, Lorenzoni (2008) identifies excessive borrowing ex ante and excessive volatility ex post. In Bianchi and Mendoza (2011), cyclical dynamics lead to a period of credit expansion up to the point where the collateral constraint becomes binding, followed by sharp decreases in credit, asset prices, and macroeconomic aggregates (see also Mendoza and Smith, 2006; Mendoza, 2010). Jeanne and Korinek (2011) study the optimal policy involved in credit booms and busts. They find that it is optimal to impose cyclical taxes to prevent agents from excessive borrowing. They emphasize that the level of the tax needs to be adjusted for the vulnerability of each sector in the economy.

In all of these papers, agents do not internalize the effect of fire sales on the value of other agents' assets, and, therefore, they overborrow ex ante. Our article differs from this literature because it is *not* a model of crisis: There are neither aggregate shocks nor multiple steady-state equilibria. The pecuniary externality is present in "normal" times, that is, in the unique steady-state equilibrium.<sup>7</sup> Furthermore, we propose a novel policy response to internalize the pecuniary externality by showing that reducing the frequency of trading can be optimal. In contrast, Jeanne and Korinek (2011) propose a Pigouvian tax on borrowing and Bianchi (2011) proposes a tax on debt to internalize the pecuniary externality. Finally, the pecuniary externality emerges from the portfolio choices and not from borrowing decisions.

## 2. THE MODEL

Time is discrete, and in each period there are three markets that open sequentially.<sup>8</sup> In the first market, agents trade money for nominal bonds. We refer to this market as the *secondary bond market*. In the second market, agents produce or consume market-2 goods. We refer to this market as the *goods market*. In the third market, agents consume and produce market-3 goods, receive money for maturing bonds, and acquire newly issued bonds. We refer to this market as the *primary bond market*. All goods are nonstorable, which means that they cannot be carried from one market to the next.

There is a  $[0, 1]$  continuum of infinitely lived agents. At the beginning of each period, agents receive two idiosyncratic i.i.d. shocks: a preference shock and an entry shock. The preference shock determines whether an agent can produce or consume market-2 goods. With probability  $1 - n$  an agent can consume but not produce, and with probability  $n$  he can produce but not consume. Consumers in the goods market are called *buyers*, and producers are called *sellers*. The entry shock determines whether agents can participate in the secondary bond market. With probability  $\pi$  they can, and with probability  $1 - \pi$  they cannot. Agents who participate in the secondary bond market are called *active*, whereas agents who do not are called *passive*. For active agents, trading in the secondary bond market is frictionless.

In the goods market, agents meet at random in bilateral meetings. We represent trading frictions by using a reduced-form matching function,  $\zeta\mathcal{M}(n, 1 - n)$ , where  $\zeta\mathcal{M}$  specifies the number of trade matches in a period and the parameter  $\zeta$  is a scaling variable, which determines the efficiency of the matching process (see, e.g., Rocheteau and Weill, 2011). We assume that the matching function has constant returns to scale and is continuous and increasing with respect to each of its arguments. Let  $\delta(n) = \zeta\mathcal{M}(n, 1 - n)(1 - n)^{-1}$  be the probability that a buyer meets a seller. The probability that a seller meets a buyer is denoted by  $\delta^s(n) = \delta(n)(1 - n)n^{-1}$ . In what

<sup>7</sup> Rojas-Breu (2013) also identifies a pecuniary externality that is present in the steady-state equilibrium. In her model, some agents use credit cards and some fiat money to acquire consumption goods. She shows that increasing the fraction of agents that use credit cards can be welfare decreasing. The intuition for this result is that increasing the fraction of agents that use credit cards can have a general equilibrium effect on the price level, which makes the agents that have no credit card worse off. This effect can be so strong that overall welfare decreases. In contrast to our model, in her model restricting the use of credit cards is a local optimum only, since it would be optimal to endow all agents with credit cards.

<sup>8</sup> Our model builds on Lagos and Wright (2005). This model is useful, because it allows us to introduce heterogeneous preferences while still keeping the distribution of asset holdings analytically tractable.

follows, to economize on notation, we suppress the argument  $n$  and refer to these probabilities as  $\delta$  and  $\delta^s$ , respectively.

In the goods market, buyers get utility  $u(q)$  from consuming  $q$  units of market-2 goods, where  $u'(q)$ ,  $-u''(q) > 0$ ,  $u'(0) = \infty$ , and  $u'(\infty) = 0$ . Sellers incur the utility cost  $c(q) = q$  from producing  $q$  units of market-2 goods.<sup>9</sup>

As in Lagos and Wright (2005), we impose assumptions that yield a degenerate distribution of portfolios at the beginning of the secondary bond market. That is, we assume that trading in the primary bond market is frictionless, that all agents can produce and consume market-3 goods, and that the production technology is linear such that  $h$  units of time produce  $h$  units of market-3 goods. The utility of consuming  $x$  units of goods is  $U(x)$ , where  $U'(x)$ ,  $-U''(x) > 0$ ,  $U'(0) = \infty$ , and  $U'(\infty) = 0$ .

Finally, agents discount between, but not within, periods. The discount factor between two consecutive periods is  $\beta = 1/(1+r)$ , where  $r > 0$  is the real interest rate.

**2.1. First-Best Allocation.** For a benchmark, it is useful to derive the planner allocation. The planner treats all agents symmetrically. His optimization problem is

$$(1) \quad \mathcal{W} = \max_{h,x,q} [\delta(1-n)u(q) - \delta^s nq] + U(x) - h,$$

subject to the feasibility constraint  $h \geq x$ . The efficient allocation satisfies  $U'(x^*) = 1$ ,  $u'(q^*) = 1$ , and  $h^* = x^*$ . These are the quantities chosen by a social planner who dictates consumption and production.<sup>10</sup>

**2.2. Pricing Mechanism.** In what follows, we study the allocations that are attainable in a market economy. To this end, we assume that the primary and secondary bond markets are characterized by perfect competition. In contrast, we will investigate several pricing mechanisms for the goods market. The baseline case is random matching and generalized Nash bargaining. However, we will also study random matching with Kalai bargaining and competitive pricing. We are in particular interested in how the different pricing mechanisms affect the portfolio choices of the agents in the primary and the secondary bond markets.

**2.3. Money and Bonds.** The description in this subsection closely follows Berentsen and Waller (2011).<sup>11</sup> There are two perfectly divisible and storable financial assets: money and one-period, nominal discount bonds. Both are intrinsically useless, since they are neither arguments of any utility function nor are they arguments of any production function. Both assets are issued by the central bank in the last market. Bonds are payable to the bearer and default-free. One bond pays off one unit of currency in the last market of the following period.

At the beginning of a period, after the idiosyncratic shocks are revealed, agents can trade bonds and money in the perfectly competitive secondary bond market. The central bank acts as the intermediary for all bond trades by recording purchases and sales of bonds. Bonds are book-keeping entries—no physical object exists. This implies that agents are not anonymous to the central bank. Nevertheless, despite having a record-keeping technology over bond trades, the central bank has no record-keeping technology over goods trades. Since agents are anonymous and cannot commit, a buyer's promise in the goods market to deliver bonds to a seller in the primary bond market is not credible.

<sup>9</sup> We assume a linear utility cost for ease of exposition. It is a simple generalization to allow for a more general convex disutility cost.

<sup>10</sup> Since our planner can dictate quantities, there is no need for either money or bonds to achieve the first-best allocation.

<sup>11</sup> Our framework is similar to Berentsen and Waller (2011). However, the questions investigated are different. The starting point in Berentsen and Waller (2011) is the observation that in monetary economies, when households face binding liquidity constraints, they can either acquire additional liquidity by selling illiquid assets or by borrowing. They show that these different methods for relaxing liquidity constraints lead to equivalent allocations under optimal policy.

Since bonds are intangible objects, they cannot be used as a medium of exchange in the goods market: hence they are illiquid. It has been shown in Kocherlakota (2003), Andolfatto (2011), and Berentsen and Waller (2011) that in environments similar to the one studied here, it is optimal that bonds are illiquid. All these papers assume unrestricted access to bond markets. One of our contributions to this literature is to show that it is not only optimal that bonds are illiquid but that it can be optimal to reduce their liquidity further by restricting access to secondary bond markets.

To motivate a role for fiat money, search models of money typically impose three assumptions on the exchange process (Shi, 2006, p. 650)<sup>12</sup>: a double coincidence problem, anonymity, and costly communication. First, our preference structure creates a single-coincidence problem in the goods market, since buyers do not have a good desired by sellers. Second, agents in the goods market are anonymous, which rules out trade credit between individual buyers and sellers. Third, there is no public communication of individual trading outcomes (public memory), which, in turn, eliminates the use of social punishments in support of gift-giving equilibria. The combination of these frictions implies that sellers require immediate compensation from buyers. In short, there must be immediate settlement with some durable asset, and money is the only such durable asset. These are the microfounded frictions that make money essential for trade in the goods market.

Denote  $M_t$  as the per capita money stock and  $B_t$  as the per capita stock of newly issued bonds at the end of period  $t$ . Then  $M_{t-1}$  ( $B_{t-1}$ ) is the beginning-of-period money (bond) stock in period  $t$ . Let  $\rho_t$  denote the price of bonds in the primary bond market. Then, the change in the money stock in period  $t$  is given by

$$(2) \quad M_t - M_{t-1} = \tau_t M_{t-1} + B_{t-1} - \rho_t B_t.$$

The change in the money supply at time  $t$  is given by three components: a lump-sum money transfer ( $T = \tau_t M_{t-1}$ ); the money created to redeem  $B_{t-1}$  units of bonds; and the money withdrawal from selling  $B_t$  units of bonds at the price  $\rho_t$ . We assume there are positive initial stocks of money  $M_0$  and bonds  $B_0$ , with  $\frac{B_0}{M_0} > \frac{n}{1-n}$ . For  $\tau_t < 0$ , the government must be able to extract money via lump-sum taxes from the economy. Let  $\mathcal{B}$  denote the bond-to-money ratio,  $B_0/M_0$ .

### 3. AGENT'S DECISIONS

For notational simplicity, the time subscript  $t$  is omitted when understood. Next-period variables are indexed by  $+1$ , and previous-period variables are indexed by  $-1$ . In what follows, we look at a representative period  $t$  and work backwards from the primary bond market (the last market) to the secondary bond market (the first market).

**3.1. Primary Bond Market.** In the primary bond market, agents can consume and produce market-3 goods. Furthermore, they receive money for maturing bonds, buy newly issued bonds, adjust their money balances by trading money for goods, and receive the lump-sum money transfer  $T$ . An agent entering the primary bond market with  $m$  units of money and  $b$  units of bonds has the indirect utility function  $V_3(m, b)$ . An agent's decision problem in the primary bond market is

$$(3) \quad \mathcal{V}_3(m, b) = \max_{x, h, m_{+1}, b_{+1}} [U(x) - h + \beta V_1(m_{+1}, b_{+1})],$$

subject to

$$(4) \quad x + \phi m_{+1} + \phi \rho b_{+1} = h + \phi m + \phi b + \phi T,$$

<sup>12</sup> See also Kocherlakota (1998), Wallace (2001), Araujo (2004), Aliprantis et al. (2007), Lagos and Wright (2008), and Araujo et al. (2012) for discussions of what makes money essential.



where  $\phi$  is the price of money in terms of market-3 goods. The first-order conditions with respect to  $m_{+1}$ ,  $b_{+1}$ , and  $x$  are  $U'(x) = 1$  and

$$(5) \quad \frac{\beta \partial V_1}{\partial m_{+1}} = \rho^{-1} \frac{\beta \partial V_1}{\partial b_{+1}} = \phi,$$

where the term  $\beta \partial V_1 / \partial m_{+1}$  ( $\beta \partial V_1 / \partial b_{+1}$ ) is the marginal benefit of taking one additional unit of money (bonds) into the next period, and  $\phi$  ( $\rho \phi$ ) is the marginal cost of doing so. Due to the quasi-linearity of preferences, the choices of  $b_{+1}$  and  $m_{+1}$  are independent of  $b$  and  $m$ . It is straightforward to show that all agents exit the primary bond market with the same portfolio of bonds and money. The envelope conditions are

$$(6) \quad \frac{\partial V_3}{\partial m} = \frac{\partial V_3}{\partial b} = \phi.$$

According to (6), the marginal value of money and bonds at the beginning of the primary bond market is equal to the price of money in terms of market-3 goods. Note that Equations (6) imply that the value function  $V_3$  is linear in  $m$  and  $b$ .

**3.2. Goods Market.** For the goods market, we make various assumptions of how the terms of trade are determined. The baseline case is random matching and generalized Nash bargaining. In the Appendix, we also derive the equilibrium under Kalai bargaining and competitive pricing.

**3.2.1. Generalized Nash bargaining.** A matched buyer and seller bargain over the terms of trade  $(q, d)$ , where  $q$  is the quantity of goods and  $d$  is the amount of money exchanged in the match. In what follows, we assume that the bargaining outcome satisfies the generalized Nash bargaining solution.

The seller's net payoff in a meeting in the goods market is given by  $-c(q) + V_3(m + d, b) - V_3(m, b)$  and the buyer's net payoff is given by  $u(q) + V_3(m - d, b) - V_3(m, b)$ . Using the linearity of  $V_3$  with respect to  $m$  and  $b$ , the bargaining problem can be formulated as follows:

$$(7) \quad \begin{aligned} (q, d) = \arg \max [u(q) - \phi d]^\theta [-c(q) + \phi d]^{1-\theta} \\ \text{s.t. } d \leq m, \end{aligned}$$

where  $\theta$  is the buyer's bargaining weight, and  $m$  is the buyer's money holding. The constraint states that the buyer cannot spend more money than the amount he brought into the match. If the buyer's constraint is nonbinding, the solution is given by  $d < m$  and  $q = q^*$ , where  $q^*$  satisfies  $u'(q^*) = 1$ . If the buyer's constraint is binding, the solution is given by  $d = m$  and

$$(8) \quad \theta u'(q) [-c(q) + \phi d] = (1 - \theta) c'(q) [u(q) - \phi d].$$

This latter condition can be written as follows:

$$(9) \quad \phi m = z(q) \equiv \frac{\theta c(q) u'(q) + (1 - \theta) u(q) c'(q)}{\theta u'(q) + (1 - \theta) c'(q)}.$$

This is by now a routine derivation of the Nash bargaining solution in a Lagos and Wright-type model. More details can be found in Lagos and Wright (2005) or Nosal and Rocheteau (2011).

3.2.2. *Value functions.* The value function of a buyer entering the goods market with  $m$  units of money and  $b$  units of bonds is

$$(10) \quad V_2^b(m, b) = \delta [u(q) + V_3(m - d, b)] + (1 - \delta) V_3(m, b).$$

With probability  $\delta$ , the buyer has a match and the terms of trade are  $(q, d)$ . Under these terms, he receives consumption utility  $u(q)$  and expected continuation utility  $V_3(m - d, b)$ . With probability  $1 - \delta$ , he has no match and receives expected continuation utility  $V_3(m, b)$ . To derive the marginal indirect utility of money and bonds, we take the total derivatives of (10) with respect to  $m$  and  $b$ , respectively, and use (6) to replace  $\frac{\partial V_3}{\partial m}$  and  $\frac{\partial V_3}{\partial b}$  to get

$$(11) \quad \frac{\partial V_2^b}{\partial m} = \delta \left[ u'(q) \frac{\partial q}{\partial m} + \phi \left( 1 - \frac{\partial d}{\partial m} \right) \right] + (1 - \delta) \phi \quad \text{and} \quad \frac{\partial V_2^b}{\partial b} = \phi.$$

If the buyer's constraint (7) is nonbinding, then  $\frac{\partial q}{\partial m} = 0$  and  $\frac{\partial d}{\partial m} = 0$ . In this case, the buyer's envelope conditions (11) satisfy  $\frac{\partial V_2^b}{\partial m} = \frac{\partial V_3}{\partial m} = \phi$ . If the constraint is binding, then  $\frac{\partial q}{\partial m} = \frac{\phi}{z'(q)}$  and  $\frac{\partial d}{\partial m} = 1$ . In this case, the buyer's envelope conditions (11) can be rewritten as follows:

$$(12) \quad \frac{\partial V_2^b}{\partial m} = \delta \phi \frac{u'(q)}{z'(q)} + (1 - \delta) \phi \quad \text{and} \quad \frac{\partial V_2^b}{\partial b} = \phi.$$

The first equality simply states that a buyer's marginal utility of money has two components: With probability  $\delta$  he has a match, and by spending the marginal unit he receives utility  $\phi u'(q) z'(q)^{-1}$ , and with probability  $1 - \delta$  he has no match, in which case by spending the marginal unit of money in the last market he receives utility  $\phi$ . The second equality states that a buyer's marginal utility of bonds at the beginning of the goods market is equal to the price of money in the last market, since bonds are illiquid in the goods market.

The value function of a seller entering the goods market with  $m$  units of money and  $b$  units of bonds is

$$(13) \quad V_2^s(m, b) = \delta^s [-c(q) + V_3(m + d, b)] + (1 - \delta^s) V_3(m, b).$$

The interpretation of (13) is similar to the interpretation of (10) and is omitted. Taking the total derivative of (13) with respect to  $m$  and  $b$ , respectively, and using (6) to replace  $\frac{\partial V_3}{\partial m}$  and  $\frac{\partial V_3}{\partial b}$  yields the seller's envelope conditions:

$$(14) \quad \frac{\partial V_2^s}{\partial m} = \frac{\partial V_2^s}{\partial b} = \phi.$$

These conditions simply state that a seller's marginal utility of money and bonds at the beginning of the goods market are equal to the price of money in the last market. The reason is that a seller has no use for these two assets in the goods market.

3.3. *Secondary Bond Market.* Let  $(\hat{m}, \hat{b})$  denote the portfolio of an active agent after trading in the secondary bond market, and let  $\varphi$  denote the price of bonds in terms of money in the secondary bond market. Then, an active agent's budget constraint satisfies

$$(15) \quad \phi m + \varphi \phi b \geq \phi \hat{m} + \varphi \phi \hat{b}.$$

The left-hand side of (15) is the sum of the real values of money and bonds with which the agent enters the secondary bond market, and the right-hand side is the real value of the portfolio with which the agent leaves the secondary bond market.



Trading is further constrained by two short-selling constraints: Active agents cannot sell more bonds, and they cannot spend more money than the amount they carry from the previous period; that is,

$$(16) \quad \hat{m} \geq 0, \quad \hat{b} \geq 0.$$

Let  $V_1^j(m, b)$  denote the value functions of an active buyer ( $j = b$ ) or an active seller ( $j = s$ ). Then, an active agent's decision problem is

$$V_1^j(m, b) = \max_{\hat{m}, \hat{b}} V_2^j(\hat{m}, \hat{b}) \quad \text{s.t.} \quad (15) \quad \text{and} \quad (16).$$

The secondary bond market's first-order conditions for active agents are

$$(17) \quad \frac{\partial V_2^j}{\partial \hat{m}} = \phi \lambda^j - \lambda_m^j, \quad \text{and} \quad \frac{\partial V_2^j}{\partial \hat{b}} = \phi \phi \lambda^j - \lambda_b^j,$$

where, for  $j = b, s$ ,  $\lambda^j$  are the Lagrange multipliers on (15) and  $\lambda_m^j$  and  $\lambda_b^j$  are the Lagrange multipliers on (16).

Finally, let  $V_1(m, b)$  denote the expected value for an agent who enters the secondary bond market with  $m$  units of money and  $b$  units of bonds before the idiosyncratic shocks are realized. Then,  $V_1(m, b)$  satisfies

$$V_1(m, b) = \pi(1 - n)V_1^b(m, b) + \pi n V_1^s(m, b) + (1 - \pi)(1 - n)V_2^b(m, b) + (1 - \pi)n V_2^s(m, b).$$

Note that passive buyers and passive sellers cannot change their portfolios and so their value functions at the beginning of the secondary bond market are  $V_2^b(m, b)$  and  $V_2^s(m, b)$ , respectively.

The envelope conditions in the secondary bond market are

$$(18) \quad \frac{\partial V_1}{\partial m} = \pi \phi [(1 - n)\lambda^b + n\lambda^s] + (1 - \pi) \left[ (1 - n) \frac{\partial V_2^b}{\partial m} + n \frac{\partial V_2^s}{\partial m} \right],$$

$$(19) \quad \frac{\partial V_1}{\partial b} = \pi \phi \phi [(1 - n)\lambda^b + n\lambda^s] + (1 - \pi) \left[ (1 - n) \frac{\partial V_2^b}{\partial b} + n \frac{\partial V_2^s}{\partial b} \right].$$

According to (18), the marginal value of money at the beginning of the period consists of four components: With probability  $(1 - n)\pi$ , an agent is an active buyer, in which case he receives the shadow value of money  $\lambda^b$ ; with probability  $n\pi$ , he is an active seller, in which case he receives the shadow value of money  $\lambda^s$ ; with probability  $(1 - n)(1 - \pi)$ , he is a passive buyer, in which case he receives the marginal value of money at the beginning of the goods market; with probability  $n(1 - \pi)$ , he is a passive seller, in which case he receives the marginal value of money at the beginning of the goods market.

#### 4. MONETARY EQUILIBRIUM

We focus on symmetric, stationary monetary equilibria, where all agents follow identical strategies and where real variables are constant over time. Let  $\eta \equiv B/B_{-1}$  denote the gross growth rate of bonds, and let  $\gamma \equiv M/M_{-1}$  denote the gross growth rate of the money supply. These definitions allow us to write (2) as follows:

$$(20) \quad \gamma - 1 - \tau = \frac{B_{-1}}{M_{-1}} (1 - \rho\eta).$$

In a stationary monetary equilibrium, the real stock of money must be constant; that is,  $\phi M = \phi_{+1} M_{+1}$ , implying that  $\gamma = \phi/\phi_{+1}$ . Furthermore, the real amount of bonds must be constant; that is,  $\phi B = \phi_{-1} B_{-1}$ . This implies  $\eta = \gamma$ , which we can use to rewrite (20) as

$$(21) \quad \gamma - 1 - \tau = \frac{B_0}{M_0} (1 - \rho\gamma).$$

The model has three types of stationary monetary equilibria. In what follows, we characterize these types of equilibria. To simplify notation, let

$$(22) \quad \Psi(q) \equiv \delta \frac{u'(q)}{z'(q)} + 1 - \delta.$$

Furthermore, in what follows we assume that  $\Psi(q)$  is decreasing in  $q$ . This assumption guarantees that our stationary monetary equilibrium derived below is unique.<sup>13</sup>

**4.1. Type I Equilibrium.** In a type I equilibrium, an active buyer's bond constraint in the secondary bond market does not bind, and a seller's cash constraint in the secondary bond market does not bind. In the Appendix, we show that a type I equilibrium can be characterized by the four equations stated in Lemma 1.

LEMMA 1. *A type I equilibrium is a time-independent list  $\{q, \hat{q}, \rho, \varphi\}$  satisfying*

$$(23) \quad \varphi = 1,$$

$$(24) \quad \frac{\gamma}{\beta} = \pi + (1 - \pi)[(1 - n)\Psi(q) + n],$$

$$(25) \quad \rho = \frac{\beta}{\gamma},$$

$$(26) \quad u'(\hat{q}) = z'(\hat{q}).$$

In a type I equilibrium, the seller's cash constraint in the secondary bond market does not bind. This can only be the case if he is indifferent between holding money or bonds, which requires  $\varphi = 1$ ; that is, that Equation (23) holds. According to (25), the price of bonds in the primary bond market is equal to its fundamental value  $\beta/\gamma$ . The reason for this result is that bonds in the primary market attain no liquidity premium (see our Discussion later), since an active buyer's constraint on bond holdings in the secondary bond market does not bind.

According to (26), active buyers consume the quantity  $\hat{q}$  that satisfies  $u'(\hat{q}) = z'(\hat{q})$ . If  $\theta < 1$ , then  $\hat{q} < q^*$ , so they consume the inefficient quantity even as  $\beta \rightarrow \gamma$ . If  $\theta = 1$ , then  $\hat{q} = q^*$ , and they consume the efficient quantity. From (24), the consumed quantity for passive buyers,  $q$ , is inefficient for all  $\theta$ .

**4.2. Type II Equilibrium.** In a type II equilibrium, an active buyer's bond constraint in the secondary bond market does not bind, and a seller's cash constraint in the secondary bond market binds. In the Appendix, we show that a type II equilibrium can be characterized by the four equations stated in Lemma 2.

<sup>13</sup> Lagos and Wright (2005, p. 472) investigate under which conditions  $\frac{u'(q)}{z'(q)}$  is decreasing in  $q$ . They argue that  $\frac{u'(q)}{z'(q)}$  "is monotone if  $\theta \approx 1$ , or if  $c(q)$  is linear and  $u'(q)$  log-concave." For a comprehensive study of existence and uniqueness of equilibrium in the Lagos and Wright framework, see Wright (2010).

LEMMA 2. *A type II equilibrium is a time-independent list  $\{q, \hat{q}, \rho, \varphi\}$  satisfying*

$$(27) \quad \frac{1}{\varphi} = \Psi(\hat{q}),$$

$$(28) \quad \frac{\gamma}{\beta} = \frac{\pi}{\varphi} + (1 - \pi)[(1 - n)\Psi(q) + n],$$

$$(29) \quad \rho = \frac{\beta}{\gamma},$$

$$(30) \quad z(q) = z(\hat{q})(1 - n).$$

The interpretations of the equilibrium equations in Lemma 2 are similar to their respective equations in Lemma 1. The key difference is that the price of bonds in the secondary bond market satisfies  $\varphi < 1$ . The reason is that now an active seller's constraint on money holdings is binding. Consequently, money is scarce and so buyers are willing to sell a fraction of their bonds at a discount; that is,  $\varphi < 1$ . Note that a buyer's constraint on bond holdings is still nonbinding, since he is only selling a fraction of his bonds. Accordingly, the price of bonds in the primary bond market,  $\rho$ , continues to be equal to its fundamental value,  $\beta/\gamma$ , as in the type I equilibrium.

Finally, (30) reflects the fact that the cash constraints of the active and passive buyers in the goods market are binding. Consequently, consumption of market-2 goods is inefficiently low for both active and passive buyers.

**4.3. Type III Equilibrium.** In a type III equilibrium, both the active buyer's bond constraint and the active seller's cash constraint in the secondary bond market bind. In the Appendix, we show that a type III equilibrium can be characterized by the four equations stated in Lemma 3.

LEMMA 3. *A type III equilibrium is a time-independent list  $\{q, \hat{q}, \rho, \varphi\}$  satisfying*

$$(31) \quad \frac{1}{\varphi} = \frac{B_0}{M_0} \frac{1 - n}{n},$$

$$(32) \quad \frac{\gamma}{\beta} = \pi[(1 - n)\Psi(\hat{q}) + n/\varphi] + (1 - \pi)[(1 - n)\Psi(q) + n],$$

$$(33) \quad \rho = \frac{\beta}{\gamma} \{1 + \pi(1 - n)[\varphi\Psi(\hat{q}) - 1]\},$$

$$(34) \quad z(q) = z(\hat{q})(1 - n).$$

According to (33), the price of bonds in the primary bond market  $\rho$  includes two components: the *fundamental value* of bonds,  $\beta/\gamma$ , and the *liquidity premium*,  $\frac{\beta}{\gamma}\pi(1 - n)[\varphi\Psi(\hat{q}) - 1]$ . The liquidity premium is increasing in  $\pi$  and equal to zero at  $\pi = 0$ . In contrast, there is no liquidity premium in the type I and type II equilibria, since an active buyer's constraint on bond holdings is not binding.

Furthermore, from (31), note that the price of bonds in the secondary bond market,  $\varphi$ , is constant (in contrast to the type II equilibrium). The reason is that in Lemma 3,  $\varphi$  is obtained from the secondary bond market budget constraint, (15). In contrast, in Lemmas 1 and 2 it is obtained from the secondary bond market first-order conditions (17). Finally, (34) has the same interpretation as (30).

TABLE 1  
BOND PRICES, EQUILIBRIUM REGIONS, AND MULTIPLIERS

Value of $\varphi$	Value of $\rho$	Inflation Range	Multipliers
$\varphi = [\Psi(\hat{q})]^{-1} = 1$	$\rho = \beta/\gamma$	$\beta \leq \gamma < \gamma_L$	$\lambda_m^s = \lambda_b^b = 0$
$\varphi = [\Psi(\hat{q})]^{-1} < 1$	$\rho = \beta/\gamma$	$\gamma_L \leq \gamma < \gamma_H$	$\lambda_m^s > \lambda_b^b = 0$
$\varphi = \frac{M_0}{B_0} \frac{n}{1-n}$	$\rho = \frac{\beta}{\gamma} \{1 + \pi(1-n)[\varphi\Psi(\hat{q}) - 1]\}$	$\gamma_H \leq \gamma$	$\lambda_m^s, \lambda_b^b > 0$

**4.4. Regions of Equilibria.** In the following proposition, we characterize three nonoverlapping regions in which these three types of equilibria exist. To this end, let  $\gamma_L$  denote the value of  $\gamma$  such that expressions (24) and (28) hold simultaneously. Furthermore, let  $\gamma_H$  denote the value of  $\gamma$  such that Equations (28) and (32) hold simultaneously. In the proof of Proposition 1, we show that such values exist and that they are unique. Furthermore, we show that  $\beta \leq \gamma_L \leq \gamma_H < \infty$ .

**PROPOSITION 1.** *If  $\beta \leq \gamma < \gamma_L$ , equilibrium prices and quantities are characterized by Lemma 1; if  $\gamma_L \leq \gamma < \gamma_H$ , they are characterized by Lemma 2; and if  $\gamma_H \leq \gamma$ , they are characterized by Lemma 3.*

Table 1 summarizes the bond prices  $\varphi$  and  $\rho$  and the relevant multipliers in the three equilibria.

In the types I and II equilibria ( $\beta \leq \gamma < \gamma_H$ ), the constraint on bond holdings of active buyers does not bind ( $\lambda_b^b = 0$ ) in the secondary bond market. This implies that the return on bonds in the secondary bond market,  $1/\varphi$ , has to be equal to the expected return on money,  $\Psi(\hat{q})$ . It also implies that the price of bonds in the primary bond market,  $\rho$ , must equal the *fundamental value* of bonds,  $\beta/\gamma$ . The economics underlying this result are straightforward. Since active buyers do not sell all their bonds for money in the secondary bond market, bonds in the primary bond market have no *liquidity premium*, and so the Fisher equation holds; that is,  $1/\rho = \gamma/\beta$ .

In contrast, in the type III equilibrium, the constraint on bond holdings of active buyers binds in the secondary bond market. Consequently, bonds attain a liquidity premium, and the Fisher equation does not hold; that is,  $1/\rho < \gamma/\beta$ .<sup>14</sup>

Figure 1 graphically characterizes the bond prices,  $\varphi$  and  $\rho$ , as a function of  $\gamma$  in the three types of equilibria. An interesting aspect of the model is that when  $\pi = 1$ , the two bond prices are equal for any value of  $\gamma$ .<sup>15</sup> Furthermore, the type I equilibrium only exists at  $\gamma = \beta$ . In contrast, there is a strictly positive spread  $\varphi - \rho$ , when  $\pi < 1$  and  $\gamma > \beta$ .

Why is there a positive spread  $\varphi - \rho$  if  $\pi < 1$ ? If  $\pi < 1$ , the price  $\rho$  reflects the fact that bonds can only be traded with probability  $\pi$  in the secondary bond market. In contrast, the price  $\varphi$  reflects the fact that active agents can trade bonds with probability 1 in the secondary bond market. Thus, the positive spread is because the bonds in the secondary bond market have a higher liquidity premium than the bonds in the primary bond market.

As can be seen in Figure 1, when  $\pi < 1$ , the price of bonds in the secondary bond market,  $\varphi$ , is constant and equal to 1 in the type I equilibrium, it is decreasing in the type II equilibrium, and it is constant in the type III equilibrium. The price of bonds in the primary bond market,  $\rho$ , follows a different pattern. In the type I and type II equilibria, it is equal to the fundamental value of bonds,  $\beta/\gamma$ , whereas in the type III equilibrium, it contains a liquidity premium. The lower  $\pi$  in the type III equilibrium is, the larger is the difference between  $\varphi$  and  $\rho$ .

<sup>14</sup> In a similar environment, Geromichalos and Herrenbrueck (2012) also analyze under what conditions a liquidity premium exists in the primary financial market.

<sup>15</sup> To see this, consider, first, Equations (32) and (33). Setting  $\pi = 1$  and rearranging yields  $\rho = \varphi = \frac{M_0}{B_0} \frac{n}{1-n}$ . Consider, next, Equations (28) and (29). Again, setting  $\pi = 1$  and rearranging yields  $\rho = \varphi = \frac{\beta}{\gamma}$ . Finally, at  $\pi = 1$ , the type I equilibrium only exists under the Friedman rule  $\gamma = \beta$ .

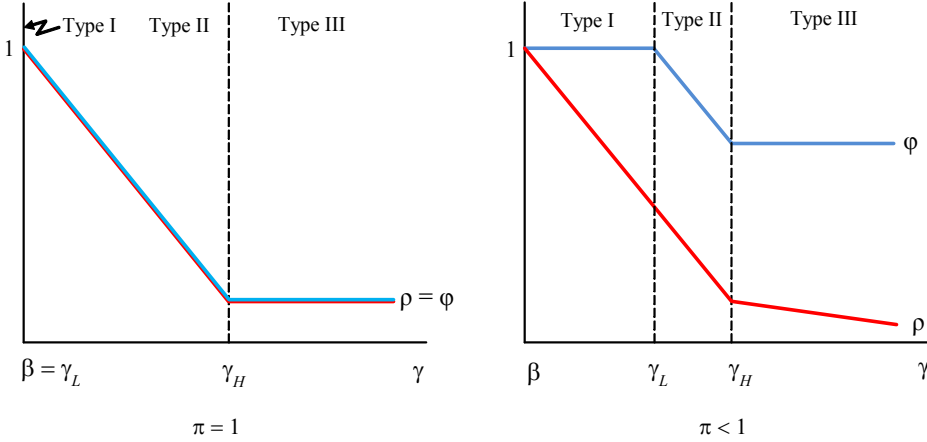


FIGURE 1

BOND PRICES WHEN  $\pi = 1$  AND  $\pi < 1$ 

## 5. OPTIMAL PARTICIPATION

In this section, we explain why restricting participation to the secondary bond market can be welfare-improving. The reason is straightforward. The secondary bond market provides insurance against the idiosyncratic liquidity shocks. At the end of a period in the primary bond market, agents choose a portfolio of bonds and money. At this point, they do not know yet whether they will be buyers or sellers in the following period. At the beginning of the following period, this information is revealed, and they can use the secondary bond market to readjust their portfolio of money and illiquid bonds.

From a welfare point of view, the benefit of the secondary bond market is that it allocates liquidity to the buyers and allows sellers to earn interest on their idle money holdings. The drawback of this opportunity is that the secondary bond market reduces the incentive to self-insure against the liquidity shocks. This lowers the demand for money in the primary bond market, which depresses its value. This effect can be so strong that it can be optimal to restrict access to the secondary bond market. The basic mechanism can be seen from the following welfare calculations.

The welfare function can be written as follows:

$$(35) \quad (1 - \beta) \mathcal{W} = (1 - n) \delta \{ \pi [u(\hat{q}) - \hat{q}] + (1 - \pi) [u(q) - q] \} + U(x^*) - x^*,$$

where the term in the curly brackets is an agent's expected period utility in the goods market, and  $U(x^*) - x^*$  is the agent's period utility in the primary bond market.

Differentiating (35) with respect to  $\pi$  yields

$$(36) \quad \frac{1 - \beta}{(1 - n) \delta} \frac{d\mathcal{W}}{d\pi} = [u(\hat{q}) - \hat{q}] - [u(q) - q] \\ + \pi [u'(\hat{q}) - 1] \frac{d\hat{q}}{d\pi} + (1 - \pi) [u'(q) - 1] \frac{dq}{d\pi}.$$

The contribution of the first two terms to the change in welfare is always positive, since in any equilibrium  $\hat{q} \geq q$  (with strict inequality for  $\gamma > \beta$ ). However, the derivatives  $\frac{d\hat{q}}{d\pi}$  and  $\frac{dq}{d\pi}$  can be negative, reflecting the fact that increasing participation reduces the incentive to self-insure

TABLE 2  
CALIBRATION TARGETS

Parameter	Target Description	Target Value
$\beta$	Average real interest rate $r$	0.991
$\gamma$	Average change in the consumer price index	1.01
$\mathcal{B}$	Average bonds-to-money ratio	3.52
$\zeta$	Set equal to 1	1.00
$\pi$	Average price of gov. bonds with a maturity of three months	0.987
$A$	Average velocity of money (annual)	6.72
$n$	Average price of gov. bonds with a remaining maturity of seven days	0.999
$\theta$	Retail sector markup	0.300

against idiosyncratic liquidity risk.<sup>16</sup> Reducing the incentive to self-insure reduces the demand for money and hence its value, which then reduces the consumption quantities  $q$  and  $\hat{q}$ .

Whether restricting participation is welfare-improving depends on which of the two effects dominates. One can show that in the type I and in the type II equilibria it is always optimal to set  $\pi = 1$ . In contrast, restricting participation in the type III equilibrium can be welfare-improving. Whether it is depends on preferences and technology. In the following, we calibrate the model to investigate whether restricting access to the secondary bond market is optimal under reasonable parameter values.<sup>17</sup>

## 6. QUANTITATIVE ANALYSIS

We choose a model period as one quarter. The functions  $u(q)$ ,  $c(q)$ , and  $U(x)$  have the forms

$$u(q) = Aq^{1-\alpha}/(1-\alpha), \quad c(q) = q, \quad \text{and} \quad U(x) = \log(x).$$

For the matching function, we follow Kiyotaki and Wright (1993) and choose

$$\mathcal{M}(B, S) = BS/(B + S),$$

where  $B = 1 - n$  is the measure of buyers and  $S = n$  the measure of sellers in the goods market. Therefore, the matching probability of a buyer in the goods market is simply given by  $\delta = \zeta \mathcal{M}(B, S)/B = \zeta n$ .

The parameters to be identified are as follows: (i) preference parameters:  $(\beta, A, \alpha)$ ; (ii) technology parameters:  $(n, \pi)$ ; (iii) bargaining power:  $\theta$ ; (iv) policy parameters: the money growth rate  $\gamma$  and the bonds-to-money ratio  $\mathcal{B}$ . Finally, we set  $\zeta = 1$  for all but one calibration, where as a robustness check we choose  $\zeta = 0.5$ .

To identify these parameters, we use U.S. data from the first quarter of 1960 to the fourth quarter of 2010. All data sources are provided in the Appendix. Table 2 lists the identification restrictions and the identified values of the parameters.

The gross growth rate of the money supply  $\gamma = 1.01$  matches the average quarterly change in the consumer price index. We set  $\beta = 0.991$  so that the real interest rate in the model matches the data, measured as the difference between the rate on AAA corporate bonds and the change in the consumer price index. The bonds-to-money ratio  $\mathcal{B} = 3.52$  matches the average empirical

<sup>16</sup> Sufficient conditions for these derivatives to be strictly negative are that inflation is sufficiently large so that the economy is in the type III equilibrium (with  $q < \hat{q} < q^*$ ) and that  $\Psi(q) = \delta \frac{u'(q)}{z'(q)} + 1 - \delta$  is decreasing in  $q$ , which is an assumption throughout the article.

<sup>17</sup> In Berentsen et al. (2011a), we provided an analytical proof that if inflation is sufficiently high, it is optimal to restrict access to the secondary bond market for  $u(q) = \ln(q)$  and perfect competition in the goods market.

TABLE 3  
NASH BARGAINING

	Description	Baseline	Markup	High $\mathcal{B}$	High $\varphi$	Low $\delta$
$A$	Utility weight	1.42	1.47	1.48	1.41	1.51
$n$	Number of sellers	0.778	0.778	0.818	0.779	0.778
$\theta$	Buyer's bargaining share	0.387	0.309	0.416	0.390	0.495
$\pi$	Calibrated $\pi$	0.588	0.550	0.594	0.574	0.514
$\pi^*$	Optimal $\pi^a$	0.589	0.548	0.568	0.555	0.532
$s_{GM}$	Goods market size	0.315	0.301	0.310	0.310	0.144

NOTES: The table displays the calibrated values for the key parameters  $A$ ,  $n$ , and  $\theta$  for the value of  $\alpha = 0.15$ . It also displays the calibrated value of  $\pi$ , the optimal value of  $\pi$  ( $\pi^*$ ), and the size of the goods market ( $s_{GM}$ ). <sup>a</sup> $\pi^*$  is calculated numerically by searching for the welfare-maximizing value of  $\pi$ , holding all other parameters at their calibrated values.

bonds-to-money ratio, which we calculate as the ratio of the total public debt to the M1 money stock.<sup>18</sup>

The parameters  $\theta$ ,  $\pi$ ,  $n$ , and  $A$  are obtained by matching the following targets simultaneously: First, we set  $\theta$  such that the markup in the goods market matches the retail data summarized by Faig and Jerez (2005). They provide a target markup of  $\mu = 0.3$  (30%).<sup>19</sup> Second, we set  $\pi$  to match the average price of government bonds with a maturity of three months, which is  $\rho = 0.987$ . Note, from Proposition 1, that  $\rho = 0.987 > \beta/\gamma = 0.982$  implies that we are in the type III equilibrium. Third, we interpret the price  $\varphi$  as the price of a government bond with a remaining maturity of seven days; that is,  $\varphi = \rho^{4/52} = 0.999$ , and we use it to calibrate  $n$ .<sup>20</sup> Fourth, we set  $A$  to match the average velocity of money. The model's velocity of money is<sup>21</sup>

$$v = \frac{Y}{\phi M_{-1}} = \frac{1 + (1 - n)\delta [\pi z(\hat{q}) + (1 - \pi) z(q)]}{z(q)},$$

which depends on  $i$  via  $q$  and  $\hat{q}$ , and on  $A$  and  $\alpha$  via the function  $z(q)$ . Although there are alternative ways to fit this relationship, we set  $A$  to match the average  $Y/\phi M_{-1}$ , using  $M1$  as our measure of money.

Our targets discussed above and summarized in Table 2 are sufficient to calibrate all but one parameter, the elasticity of the utility function  $\alpha$ . Berentsen et al. (2011b) estimate that  $\alpha \in (0.105, 0.211)$ , depending on the calibration method. We, therefore, first present the calibration results for an average value of  $\alpha = 0.15$  and, then, show the effects of different values of  $\alpha$  later on.<sup>22</sup>

**6.1. Baseline Results and Robustness Checks under Nash Bargaining.** Table 3 presents the results for the baseline calibration and four robustness checks under generalized Nash bargaining. The robustness checks are defined as follows: In the calibration labeled “markup,” we

<sup>18</sup> This definition is in line with Martin (2012).  
<sup>19</sup> See Aruoba et al. (2011) or Berentsen et al. (2011b) on calibrating LW-type models, including matching the markup data.

<sup>20</sup> We show in the robustness analysis that our results are not very sensitive to the choice of  $\varphi$ .  
<sup>21</sup> The real output in the goods market is  $Y_{GM} = (1 - n)\delta[\pi\phi\hat{m} + (1 - \pi)\phi m]$ , where  $\phi\hat{m} = z(\hat{q})$  and  $\phi M_{-1} = \phi m = z(q)$ , and the real output in the primary bond market is  $Y_{PBM} = 1$ . Accordingly, total real output of the economy adds up to  $Y = Y_{GM} + Y_{PBM}$ , and the model-implied velocity of money is  $v = Y/\phi M_{-1}$ .

<sup>22</sup> Most monetary models that calibrate variants of the Lagos and Wright (2005) framework set  $\alpha$  to match the elasticity of money demand with respect to the nominal interest rate. We cannot do this because in our framework the interest rate on  $\rho$  represents the yield on three-month government bonds, whereas related studies work with the AAA Moody's corporate bond yield to calculate the elasticity of money demand. Using U.S. data from 1960 to 2010, we obtain an empirical elasticity of money demand with respect to the yield on three-month government bonds of  $\xi_{gov} = 0.05$ . The elasticity of money demand in our model is negative by construction, which precludes the use of this target.



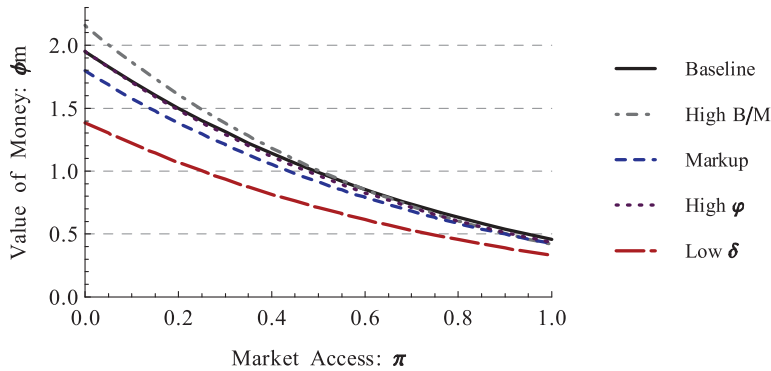


FIGURE 2  
 $\phi m$  FOR DIFFERENT VALUES OF  $\pi$

target a markup in the goods market of 40% instead of 30%; in the calibration labeled “high  $B$ ,” we target  $B = 4.5$  instead of  $B = 3.5$ ; in the calibration labeled “high  $\varphi$ ,” we target a remaining maturity of government bonds of one day instead of seven days; and in the calibration labeled “low  $\delta$ ,” we set  $\zeta = 0.5$  instead of  $\zeta = 1$ .

Table 3 presents the key parameter values for the baseline calibration and the robustness checks when  $\alpha = 0.15$ . To address the question of whether there is too much trading in the secondary bond market, we also calculate the optimal entry probability  $\pi^*$  for each case. It is calculated as follows. For each set of calibrated parameter values, we numerically search for the value of  $\pi$  that maximizes ex ante welfare, defined by (35).

We find two key results. First, our calibrations always yield an entry probability  $\pi$ , which is strictly below 1. Second, the optimal entry probability  $\pi^*$  is below the calibrated entry probability for a sufficiently high markup, a high bonds-to-money ratio, and a high value of  $\varphi$ . In contrast, under the baseline calibration and the calibration with a low matching probability  $\delta$ , we find that  $\pi^*$  is above  $\pi$ . In Table 3, we also provide the estimates of the model-implied goods market share,  $s_{GM} = Y_{GM}/Y$ . Under Nash bargaining, it is approximately 31% for  $\zeta = 1$ , and for a lower matching probability ( $\zeta = 0.5$ ) it is about 14%, which is in line with the estimates of Berentsen et al. (2011b) and related studies.

To provide more intuition about why  $\pi^*$  is smaller than the calibrated value of  $\pi$  for most robustness checks, we show, in Figure 2, the effect of changing  $\pi$  on the value of money  $\phi m$ .

For all cases, the value of money in Figure 2 is strictly decreasing in  $\pi$ . This confirms our intuition that having access to the secondary bond market, reduces the demand for money,  $m$ , and since the supply of money is given and equal to  $M$ , it reduces the real stock of money  $\phi m$ . For a high bonds-to-money ratio, the value of money decreases at a faster rate. This is intuitive, as a high value of  $B$  allows active agents to trade more bonds in the secondary bond market and, thus, reduces the incentive to self-insure against the liquidity shocks.

How sensitive are our results to the gross inflation rate  $\gamma$ ? To answer this question, we show the effect of increasing  $\gamma$  on the difference between  $\pi^*$  and  $\pi$  in Figure 3.

Figure 3 shows that for all the calibration experiments an increase in  $\gamma$  lowers the difference  $\pi^* - \pi$ . The explanation for this result is straightforward. A higher inflation rate increases the opportunity cost of holding money and, thus, reduces the incentive to self-insure against the liquidity shocks. This reduces the value of money and so welfare. To correct this pecuniary externality effect, a substantially lower value of  $\pi^*$  is needed to induce agents to increase their money holdings.

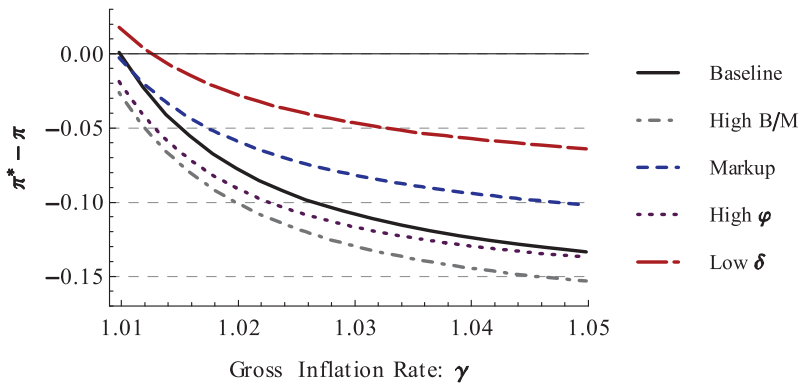


FIGURE 3  
 $\pi^* - \pi$  FOR INCREASING VALUES OF  $\gamma$

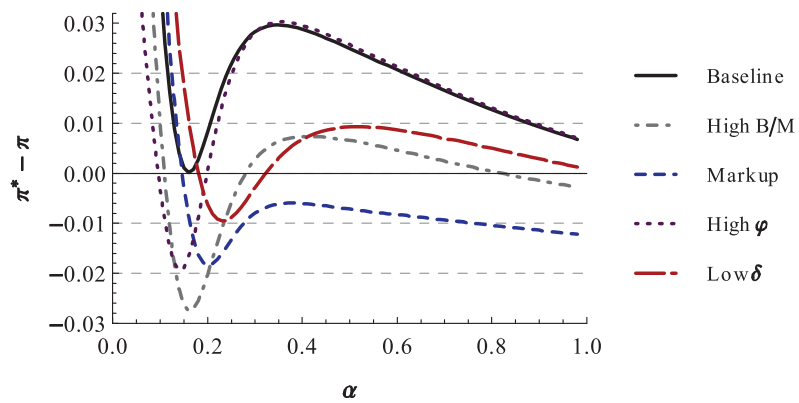


FIGURE 4  
 $\pi^* - \pi$  FOR DIFFERENT VALUES OF  $\alpha$

6.2. *The Effect of the Elasticity of the Utility Function.* How sensitive are our results to the choice of  $\alpha$ ? In order to answer this question, we recalibrate each model presented in Table 3 for different values of  $\alpha \in (0, 1)$  and draw the difference  $\pi^* - \pi$  in Figure 4.

For the baseline calibration under Nash bargaining, we find that  $\pi < \pi^*$  for any  $\alpha \in (0, 1)$ . In contrast, for calibrations “markup,” “high  $B$ ,” “high  $\varphi$ ,” and “low  $\delta$ ” there is a strictly positive range for which there is too much entry; that is,  $\pi > \pi^*$ . A higher markup in the goods market appears to have the largest effect, since  $\pi^* - \pi < 0$  for any value of  $\alpha > 0.14$ .

Note also that increasing the bonds-to-money ratio from 3.5 to 4.5 results in  $\pi^* - \pi < 0$  for  $0.1 < \alpha < 0.28$  and  $0.82 < \alpha < 1$ . This is insofar interesting, since in the U.S. data the bonds-to-money ratio is steadily increasing over time in our sample, and since 1996 it is above the value of 4.5. Furthermore, in 2010 it reached 7.7.

6.3. *Other Pricing Mechanisms.* Hereafter, we compare the calibration results for different trading protocols to our baseline calibration under Nash bargaining. The calibration labeled “Kalai” refers to Kalai bargaining, where we use  $z^K(q)$  instead of  $z(q)$ . The calibration labeled “CP” refers to competitive pricing. By setting  $\theta = 1$ , the model equations reduce to the ones that one obtains from assuming competitive pricing in the goods market. See the Appendix, where

TABLE 4  
OTHER PRICING MECHANISMS

	Description	Baseline	Kalai	CP
$A$	goods market utility weight	1.42	1.38	1.30
$n$	number of sellers	0.778	0.778	0.778
$\theta$	buyer's bargaining power	0.387	0.433	1
$\pi$	calibrated $\pi$	0.588	0.564	0.772
$\pi^*$	optimal $\pi^a$	0.589	0.659	0.949
$s_{GM}$	goods market size	0.315	0.306	0.382

NOTES: The table displays the calibrated values for the key parameters  $A$ ,  $n$ , and  $\theta$  for the value of  $\alpha = 0.15$ . It also displays the calibrated value of  $\pi$ , the optimal value of  $\pi$  ( $\pi^*$ ), and the size of the goods market ( $s_{GM}$ ). <sup>a</sup> $\pi^*$  is calculated numerically by searching for the welfare maximizing value of  $\pi$ , holding all other parameters at their calibrated values.

we also derive the Kalai bargaining solution. Table 4 presents the parameter values obtained for the different pricing mechanisms for  $\alpha = 0.15$ .<sup>23</sup>

In contrast to Nash bargaining, for competitive pricing the difference of  $\pi^* - \pi$  is clearly positive. This indicates that the access to the secondary bond market is too low. Note though, that the result that  $\pi^* < 1$  continues to hold. That is, it is not optimal to grant unrestricted access to the secondary bond market. Our reading of these results is that our calibration measures the frictions in these markets and that under certain calibrations the existing frictions  $\pi$  are too high ( $\pi < \pi^*$ ) and in others they are too low ( $\pi > \pi^*$ ). However, we always find that eliminating all frictions by setting  $\pi = 1$  is suboptimal.

## 7. DISCUSSION

In the discussion, we first relate our framework to Kocherlakota (2003) and Shi (2008). We then show that if agents have a choice to participate in the secondary bond market, they strictly prefer to do so.

**7.1. Relation to Kocherlakota (2003) and Shi (2008).** In the Shi (2008) framework, there is no secondary bond market. Rather, he assumes that agents are allowed to use bonds and money to pay for goods in some trade meetings, while they can only use money to pay for goods in some other trade meetings.<sup>24</sup> He shows that such a legal restriction can be welfare-improving. In his model, there are two types of goods: red and green. The costs of production are the same for the two colors, but the marginal utilities differ. Formally, consumption utility is represented by the utility function  $\theta^j u(c^j)$ , where  $c^j$  is consumption of good  $j$ , and  $j$  denotes the good type:  $G$  (green) and  $R$  (red). It is assumed that  $\theta^G = 1$  and  $\theta^R = \theta > 0$ .

Once agents are matched, they receive a *matching shock*: With a 50% probability the red good is produced, and with a 50% probability the green good is produced. In each match, buyers make a take-it-or-leave-it offer. The legal restriction imposed by Shi (2008) is that while money can be used in both matches, bonds as a means of payment cannot be used in red trades. This legal restriction is welfare-improving if the relative marginal utility of red goods is less than one, but not too small. The intuition is that in the economy without this legal restriction agents consume the same amount of goods in all matches, because money and bonds are perfect substitutes (see Table 5, where  $q_1^R = q_1^G$ ). This allocation is inefficient, because efficiency requires that consumption of green goods is higher than consumption of red goods (see Table 5, where the efficient quantities satisfy  $q_*^R < q_*^G$ ). The legal restriction, thus, shifts consumption from the red

<sup>23</sup> Notice that the markup target is only used for the baseline calibration and Kalai bargaining.

<sup>24</sup> Since in the Shi framework agents can use money and bonds to pay for goods in some matches, it is more closely related to the literature that studies competing media of exchange (see, e.g., Geromichalos et al., 2007; Lagos and Rocheteau, 2007, 2008, 2009; Lester et al., 2012).

TABLE 5  
CONSUMED QUANTITY IN SHI, KOCHERLAKOTA, AND OUR MODEL

Case <sup>a</sup>	Description	Shi (IS) <sup>b</sup>	Kocherlakota	Our Model <sup>c</sup>
1	Liquid bonds	$q_1^G = q_1^R < q_*^R < q_*^G$	$q_1 < q_*$	$q_1 < q_*$
2	Illiquid bonds, SBM, $\pi = 1$	—	$q_1 < q_2 < q_*$	$q_1 < q_2 < q_*$
3	Partially liquid bonds (Shi)	$q_3^R < q_1^R = q_1^G < q_3^G$	—	—
4	Illiquid bonds, SBM, $\pi < 1$	—	—	$q_4^{NA} < q_4^A < q_*$

NOTES: Variables labeled with an \* indicate efficient quantities. In Shi, the efficient quantities differ in green matches,  $q_*^G$ , and red matches,  $q_*^R$ , whereas in our model there is only one efficient quantity,  $q_*$ .

<sup>a</sup>The lower-case index of the variables refers to the equilibrium quantities in the following cases: (1) an economy with liquid bonds; (2) an economy with illiquid bonds and with unrestricted access to the secondary bond market (SBM); (3) an economy with partially liquid bonds without an SBM; (4) an economy with illiquid bonds and with restricted access to the secondary bond market.

<sup>b</sup>IS refers to the “Imperfect Substitutability” equilibrium in Shi.

<sup>c</sup> $q_4^A$  and  $q_4^{NA}$  are the consumption quantities for agents that have access and no access to the SBM in our model.

good to the more highly valued green good. This smoothes marginal utilities across green and red matches, which is welfare increasing.

In Shi, the welfare improvement arises because the legal restriction shifts consumption toward the more desired green good. In Kocherlakota and in our model, this mechanism is absent, since there is only one good and, hence, only one efficient quantity (see Table 5, where the efficient quantity is denoted  $q_*$ ). The welfare benefits of creating illiquid bonds and letting agents trade them in a secondary bond market arises because it increases consumption as explained later.

When bonds are as liquid as money, all agents consume the same quantity  $q_1 < q_*$  in Kocherlakota and in our model, and  $q_1^G = q_1^R$  in Shi (see Table 5, case 1).<sup>25</sup> When bonds are illiquid and when there is a secondary bonds market, where all agents can trade ( $\pi = 1$ ), all agents consume the same quantity as indicated by the quantity  $q_2 < q_*$  in case 2 of Table 5. Note, though, that  $q_1 < q_2$  implies that creating illiquid bonds and allowing agents to trade them for money in response to liquidity shocks in a secondary bond market improves the allocation. We next consider our key result, which is that it can be welfare-improving to set  $\pi < 1$ . There are two effects of such a policy. First, it introduces variance in the marginal utilities across matches, since agents who have access to the secondary bonds market trade different quantities than agents who have no access. In Table 5, case 4, the former trade the quantity  $q_4^A$ , and the latter trade the quantity  $q_4^{NA}$ , with  $q_4^{NA} < q_4^A$ . Introducing consumption variability is clearly costly. Nevertheless, we show that it can be welfare-improving because such a policy can increase the value of money and, hence, consumption quantities. Since our mechanism adds a wedge between marginal utilities across matches, whereas Shi’s mechanism reduces such a wedge, it should be clear that our mechanism is very different from Shi’s legal restriction model.

**7.2. Endogenous Participation.** So far, we have assumed that participation in the secondary bond market is determined by the exogenous idiosyncratic participation shock  $\pi$ . Suppose instead that each agent has a choice. Recall that  $V_1^b(m, b)$  is the expected lifetime utility of a buyer at the beginning of the secondary bond market, and  $V_2^b(m, b)$  is the expected lifetime utility of a buyer at the beginning of the goods market who had no access to the secondary bond market. Then, for a buyer, it is optimal to participate if

$$V_1^b(m, b) \geq V_2^b(m, b).$$

Note that the exact experiment here is to keep all prices at their equilibrium values for a given participation rate  $\pi$  and, then, to ask the question whether a single buyer would prefer to enter

<sup>25</sup> In fact,  $q_1$  is the same quantity that would be consumed in a model without bonds (e.g., Lagos and Wright, 2005) or in a model with liquid bonds, since liquid bonds do not affect the allocation.

the secondary bond market. The move of a single buyer from passive to active does not change equilibrium prices.

LEMMA 4. *In any equilibrium,  $V_1^b(m, b) - V_2^b(m, b) \geq 0$ .*

According to Lemma 4, a buyer is always better off when participating in the secondary bond market. To develop an intuition for this result, note that, as shown in the proof of Lemma 4,

$$(37) \quad V_1^b(m, b) - V_2^b(m, b) = u(\hat{q}) - \hat{q} - [u(q) - q] - i(\hat{q} - q),$$

where  $i = (1 - \varphi)/\varphi$  is the nominal interest rate. A passive buyer's period surplus is  $u(q) - q$ , whereas an active buyer's surplus is  $u(\hat{q}) - \hat{q} - i(\hat{q} - q)$ , where the term  $i(\hat{q} - q)$  measures the utility cost of selling bonds to finance the difference  $\hat{q} - q \geq 0$ . The difference  $u(\hat{q}) - \hat{q} - [u(q) - q]$  is strictly positive, whereas the term  $-i(\hat{q} - q)$  is negative. The reason is that in any equilibrium,  $q \leq \hat{q} \leq q^*$ . Thus, the equilibrium interest rate cannot be too large in order for (37) to be positive. In the proof of Lemma 4, we replace  $i$  in (37) for all three types of equilibria and find that  $V_1^b(m, b) - V_2^b(m, b) > 0$ .

We now turn to the sellers. For them, we also find that they are better off when participating in the secondary bond market.

LEMMA 5. *In any equilibrium,  $V_1^s(m, b) - V_2^s(m, b) \geq 0$ .*

In the type I equilibrium, the nominal interest rate is  $i = 0$ . In this case,  $V_1^s(m, b) = V_2^s(m, b)$ . In the type II and type III equilibria, the nominal interest rate is  $i > 0$ . In this case, the seller strictly prefers to enter, since  $V_1^s(m, b) > V_2^s(m, b)$ .

## 8. CONCLUSION

We construct a general equilibrium model with a liquid asset and an illiquid asset. Agents experience idiosyncratic liquidity shocks after which they can trade these assets in a secondary bond market. We find that an agent's portfolio choice of liquid and illiquid assets involves a pecuniary externality. An agent does not take into account that by holding more of the liquid asset he not only acquires additional insurance against his own idiosyncratic liquidity risks, but he also marginally increases the value of the liquid asset, which improves insurance for other market participants. This pecuniary externality can be corrected by restricting, but not eliminating, access to the secondary bond market.

Our results provide justifications for policies that make trading in financial markets difficult. This should come as no shock to anyone who knows the theory of the second best. Our model is an incomplete market model. It is well known that in such environments, adding frictions and/or taxes can improve the allocation.

## APPENDIX

### A. PROOFS

PROOF OF LEMMA 1. We first note that in *any* equilibrium (i.e., types I, II, and III), a buyer will never use all his money to buy bonds in the secondary bond market, implying that  $\lambda_m^b = 0$ . Furthermore, a seller will never spend all his bonds for money in the secondary bond market, implying that  $\lambda_b^s = 0$ .

Furthermore, in a type I equilibrium, an active buyer's bond constraint in the secondary bond market does not bind ( $\lambda_b^b = 0$ ), and a seller's cash constraint in the secondary bond market does

not bind ( $\lambda_m^s = 0$ ). Using these values for the multipliers, we can rewrite the secondary bond market first-order conditions (17) as follows:

$$(A.1) \quad \frac{\partial V_2^b}{\partial \hat{m}} = \phi \lambda^b \quad \text{and} \quad \frac{\partial V_2^b}{\partial \hat{b}} = \varphi \phi \lambda^b,$$

$$(A.2) \quad \frac{\partial V_2^s}{\partial \hat{m}} = \phi \lambda^s \quad \text{and} \quad \frac{\partial V_2^s}{\partial \hat{b}} = \varphi \phi \lambda^s.$$

Furthermore, combining the previous expressions with (12) and (14), we have

$$(A.3) \quad \lambda^b = \delta \frac{u'(\hat{q})}{z'(\hat{q})} + 1 - \delta \quad \text{and} \quad \varphi \lambda^b = 1,$$

$$(A.4) \quad \lambda^s = 1 \quad \text{and} \quad \varphi \lambda^s = 1.$$

Then, (A.4) implies that  $\varphi = 1$ ; that is, that (23) holds.

Then, from (A.3), the fact that  $\varphi = 1$  immediately implies that  $\lambda^b = 1$ , which then implies that  $u'(\hat{q}) = z'(\hat{q})$ ; that is, that (26) holds.

Use (12) and (14) to write (18) and (19) as follows:

$$(A.5) \quad \frac{\partial V_1}{\partial m} = \pi \phi [(1 - n) \lambda^b + n \lambda^s] + (1 - \pi) [(1 - n) \phi \Psi(q) + n \phi],$$

$$(A.6) \quad \frac{\partial V_1}{\partial b} = \pi \phi [(1 - n) \varphi \lambda^b + n \varphi \lambda^s] + (1 - \pi) [(1 - n) \phi + n \phi].$$

Use the primary bond market first-order conditions (5) to write the previous equations as follows:

$$(A.7) \quad \frac{\gamma}{\beta} = \pi [(1 - n) \lambda^b + n \lambda^s] + (1 - \pi) [(1 - n) \Psi(q) + n],$$

$$(A.8) \quad \frac{\rho \gamma}{\beta} = \pi \phi [(1 - n) \lambda^b + n \lambda^s] + 1 - \pi.$$

We have already established that in the type I equilibrium  $\lambda^b = \lambda^s = \varphi = 1$ . This implies, from (A.8), that  $\rho = \beta/\gamma$ ; that is, that Equation (25) holds. Finally, (24) immediately follows from (A.7).

Note that if  $\theta < 1$ , active buyers consume the inefficient quantity, since  $\hat{q} < q^*$  even as  $\beta \rightarrow \gamma$ . If  $\theta = 1$ ,  $u'(\hat{q}) = 1$ , so they consume the efficient quantity  $\hat{q} = q^*$ . ■

**PROOF OF LEMMA 2.** We first show that Equation (30) holds. In the type II equilibrium, all buyers spend all their money in the goods market. Consequently,  $z(q) = \phi m$  and  $z(\hat{q}) = \phi \hat{m}$  hold. The last two equations imply

$$(A.9) \quad z(q) = z(\hat{q})m/\hat{m}.$$

Each active buyer exits the secondary bond market with  $\hat{m}$  units of money, whereas an active seller exits with zero units of money. A passive agent (a seller or a buyer) exits the secondary

bond market with  $m$  units of money; therefore  $M_{-1} = (1 - n)\pi\hat{m} + n\pi \times 0 + (1 - \pi)m$ . Replacing  $m = M_{-1}$ , we get  $\hat{m} = M_{-1}/(1 - n)$ . Use  $\hat{m} = M_{-1}/(1 - n)$  and  $m = M_{-1}$  to replace  $\hat{m}$  and  $m$  in (A.9), respectively, to get  $z(q) = z(\hat{q})(1 - n)$ ; that is, Equation (30) holds.

We now show that (27)–(29) hold. As argued in the proof of Lemma 1,  $\lambda_m^b = 0$  and  $\lambda_b^s = 0$  in any equilibrium. In a type II equilibrium, an active buyer's bond constraint in the secondary bond market does not bind, that is,  $\lambda_b^b = 0$ , and a seller's cash constraint in the secondary bond market binds, that is,  $\lambda_m^s > 0$ . Using these values for the multipliers, the secondary bond market first-order conditions (17) can be rewritten as follows:

$$(A.10) \quad \frac{\partial V_2^b}{\partial \hat{m}} = \phi\lambda^b \quad \text{and} \quad \frac{\partial V_2^b}{\partial \hat{b}} = \phi\phi\lambda^b,$$

$$(A.11) \quad \frac{\partial V_2^s}{\partial \hat{m}} = \phi\lambda^s - \lambda_m^s \quad \text{and} \quad \frac{\partial V_2^s}{\partial \hat{b}} = \phi\phi\lambda^s.$$

Using the previous expressions in (12) and (14), we obtain

$$(A.12) \quad \lambda^b = \delta \frac{u'(\hat{q})}{z'(\hat{q})} + 1 - \delta \quad \text{and} \quad \phi\lambda^b = 1,$$

$$(A.13) \quad \lambda_m^s = \phi(\lambda^s - 1) \quad \text{and} \quad \phi\lambda^s = 1.$$

From (A.13),  $\lambda_m^s = \phi(\lambda^s - 1) = \phi(\frac{1}{\phi} - 1)$ . Note that  $\lambda_m^s > 0$  implies  $\phi < 1$ .

Expression (27) follows directly from (A.12). As in Lemma 1, use (12) and (14) to write (18) and (19) as follows:

$$(A.14) \quad \frac{\partial V_1}{\partial m} = \pi\phi[(1 - n)\lambda^b + n\lambda^s] + (1 - \pi)[(1 - n)\phi\Psi(q) + n\phi],$$

$$(A.15) \quad \frac{\partial V_1}{\partial b} = \pi\phi[(1 - n)\phi\lambda^b + n\phi\lambda^s] + (1 - \pi)[(1 - n)\phi + n\phi].$$

Use the primary bond market first-order conditions (5) to write the previous equations as follows:

$$(A.16) \quad \frac{\gamma}{\beta} = \pi[(1 - n)\lambda^b + n\lambda^s] + (1 - \pi)[(1 - n)\Psi(q) + n],$$

$$(A.17) \quad \frac{\rho\gamma}{\beta} = \pi\phi[(1 - n)\lambda^b + n\lambda^s] + 1 - \pi.$$

Substituting  $\lambda^b$  and  $\lambda^s$  in (A.17) yields  $\rho = \beta/\gamma$ ; that is, Equation (29) holds. Finally, (28) immediately follows from (A.16). ■

**PROOF OF LEMMA 3.** The proof that Equation (34) holds in a type III equilibrium follows the proof that Equation (30) holds in Lemma 2, and is not repeated here.

We next show that Equation (31) holds. An active agent enters the secondary bond market with a real portfolio  $\phi m + \phi\phi b$  of money and bonds. As a buyer, he sells all his bonds in the type III equilibrium, and thus he exits the secondary bond market with a portfolio  $\phi\hat{m}$ . As a seller, he sells all his money and thus exits this market with a portfolio  $\phi\phi\hat{b}$ . Therefore  $\phi m + \phi\phi b = \phi\hat{m}$



holds for an active buyer, and  $\phi m + \phi \phi b = \phi \phi \hat{b}$  holds for an active seller. Combining the two equations yields

$$(A.18) \quad \hat{m} = \phi \hat{b}.$$

Immediately after the secondary bond market closes, but before the goods market opens, the stock of money in circulation is in the hands of active buyers and passive agents (sellers and buyers). Active sellers hold no money at the end of the secondary bond market. Consequently,  $M_{-1} = \pi(1 - n)\hat{m} + \pi n \times 0 + (1 - \pi)m$ . Eliminate  $m$ , using  $m = M_{-1}$ , and rearrange to get

$$(A.19) \quad \hat{m} = \frac{M_{-1}}{1 - n}.$$

The stock of bonds in circulation is in the hands of active sellers and passive agents (sellers and buyers), whereas active buyers hold no bonds at the end of the secondary bond market. Thus, the stock of bonds is equal to  $B_{-1} = \pi(1 - n) \times 0 + \pi n \hat{b} + (1 - \pi)b$ . Since passive agents do not trade in the secondary bond market, they enter the goods market with the same amount of bonds they had at the beginning of the period,  $b = B_{-1}$ . Use this equation to eliminate  $b$  in the bond stock expression above and get

$$(A.20) \quad \hat{b} = \frac{B_{-1}}{n}.$$

Replace  $\hat{m}$  and  $\hat{b}$  in (A.18) by using (A.19) and (A.20), respectively. Since the bonds-to-money ratio is constant over time, we can replace the time  $t - 1$  stock of money and bonds with their respective initial values. Equation (31) then follows.

Finally, we show that (32) and (33) hold. In any equilibrium,  $\lambda_m^b = 0$  and  $\lambda_b^s = 0$ . In a type III equilibrium, a seller's cash constraint in the secondary bond market binds, that is,  $\lambda_m^s > 0$ , and a buyer's bond constraint in the secondary bond market binds; that is,  $\lambda_b^b > 0$ . Using these multipliers, the secondary bond market first-order conditions (17) become

$$(A.21) \quad \frac{\partial V_2^b}{\partial \hat{m}} = \phi \lambda^b \quad \text{and} \quad \frac{\partial V_2^b}{\partial \hat{b}} = \phi \phi \lambda^b - \lambda_b^b,$$

$$(A.22) \quad \frac{\partial V_2^s}{\partial \hat{m}} = \phi \lambda^s - \lambda_m^s \quad \text{and} \quad \frac{\partial V_2^s}{\partial \hat{b}} = \phi \phi \lambda^s.$$

Using the previous expressions in (12) and (14), we obtain

$$(A.23) \quad \lambda^b = \delta \frac{u'(\hat{q})}{z'(\hat{q})} + (1 - \delta) \quad \text{and} \quad \lambda_b^b = \phi (\phi \lambda^b - 1),$$

$$(A.24) \quad \lambda_m^s = \phi (\lambda^s - 1) \quad \text{and} \quad \phi \lambda^s = 1.$$

Like in a type II equilibrium,  $\lambda_m^s = \phi (\lambda^s - 1) = \phi (\frac{1}{\phi} - 1)$ , and since  $\lambda_m^s > 0$ , then  $\phi < 1$ . Unlike in a type II equilibrium, from (A.21), we find  $\lambda_b^b = \phi (\phi \lambda^b - 1) = \phi [\phi \Psi(\hat{q}) - 1]$ . Since  $\lambda_b^b > 0$ ,  $\Psi(\hat{q}) > 1/\phi$ , and so (27) does not hold in a type III equilibrium.

Use (12) and (14) to write (18) and (19) as follows:

$$(A.25) \quad \frac{\partial V_1}{\partial m} = \pi \phi [(1 - n)\lambda^b + n\lambda^s] + (1 - \pi) [(1 - n)\phi \Psi(q) + n\phi],$$

$$(A.26) \quad \frac{\partial V_1}{\partial b} = \pi\phi[(1-n)\varphi\lambda^b + n\varphi\lambda^s] + (1-\pi)[(1-n)\phi + n\phi].$$

Using the primary bond market first-order conditions (5), the previous equations can be rewritten as follows:

$$(A.27) \quad \frac{\gamma}{\beta} = \pi[(1-n)\lambda^b + n\lambda^s] + (1-\pi)[(1-n)\Psi(q) + n],$$

$$(A.28) \quad \frac{\rho\gamma}{\beta} = \pi\varphi[(1-n)\lambda^b + n\lambda^s] + 1 - \pi.$$

Substituting  $\lambda^b$  and  $\lambda^s$  in (A.27) and (A.28) yields (32) and (33), respectively. ■

**PROOF OF PROPOSITION 1.** The critical values  $\gamma_L$  and  $\gamma_H$  exist and are unique, since  $\Psi(q)$  is decreasing in  $q$  by assumption. We now show that  $\beta \leq \gamma_L \leq \gamma_H < \infty$ . To do so, first note that  $\gamma_L$  satisfies (24), that is,

$$\frac{\gamma_L}{\beta} = \pi + (1-\pi)[(1-n)\Psi(q) + n],$$

at the intersection between type I and type II equilibria. Also note that  $\beta > \gamma_L \Leftrightarrow \Psi(q) < 1 \Leftrightarrow u'(q) < z'(q)$  implies that passive agents consume more than the efficient quantity,  $q > q^*$ . This is clearly not an equilibrium since they can be better off by reducing consumption,  $q$ . Moreover,  $\beta = \gamma_L \Leftrightarrow \Psi(q) = 1 \Leftrightarrow u'(q) = z'(q)$ , which means that passive agents consume the efficient quantity,  $q = q^*$ . Finally,  $\beta < \gamma_L \Leftrightarrow \Psi(q) > 1 \Leftrightarrow u'(q) > z'(q)$ , which is also an equilibrium, since passive agents would like to consume more but do not have enough money to do so. Hence,  $\beta \leq \gamma_L$  must hold in any equilibrium. Now, note that  $\gamma_H$  satisfies (28), that is,

$$\frac{\gamma_H}{\beta} = \pi\Psi(\hat{q}) + (1-\pi)[(1-n)\Psi(q) + n],$$

at the intersection between type II and type III equilibria. Also note that  $\gamma_H \nearrow \infty$  iff  $\hat{q}, q \searrow 0$ . Since consumed quantities are always strictly positive, then  $\gamma_H < \infty$ . Finally, note that the only sequence of equilibria can be types I, II, and III, that is,  $\gamma_L \leq \gamma_H$ . Thus, we have shown that  $\beta \leq \gamma_L \leq \gamma_H < \infty$ .

*Derivation of  $\gamma_L$ :* The critical value  $\gamma_L$  is the value of  $\gamma$  such that expressions (24) and (28) hold simultaneously; that is, such that  $\Psi(\hat{q}) = 1$ . Such a value exists and is unique, since we assume that  $\Psi(q)$  is decreasing in  $q$ .

*Derivation of  $\gamma_H$ :* The critical value  $\gamma_H$  is the value of  $\gamma$  such that Equations (28) and (32) hold simultaneously; that is, such that  $\Psi(\hat{q}) = \frac{B_0}{M_0} \frac{1-n}{n} > 1$ . Again, such a value exists and is unique, since we assume that  $\Psi(q)$  is decreasing in  $q$ . ■

**PROOF OF LEMMA 4.** From the buyer's problem in the secondary bond market,  $V_1^b(m, b) = V_2^b(\hat{m}, \hat{b})$ , where  $\hat{m}$  and  $\hat{b}$  are the quantities of money and bonds that maximize  $V_2^b$ . In any equilibrium, the buyer's budget constraint (15) holds with equality. Thus, we can use (15) to eliminate  $\hat{b}$  from  $V_2^b(\hat{m}, \hat{b})$  and get

$$(A.29) \quad V_1^b(m, b) = V_2^b\left(\hat{m}, \frac{\phi m + \phi\phi b - \phi\hat{m}}{\phi\phi}\right).$$

Next, use (4), (10), and (3), to get

$$V_1^b(m, b) = \delta \{u[q(\hat{m})] - \phi d(\hat{m})\} + U(x^*) - x^* + \phi\hat{m} + \phi T - \phi m_{+1}$$

$$(A.30) \quad + \frac{\phi m + \phi \phi b - \phi \hat{m}}{\phi} - \phi \rho b_{+1} + \beta V_1(m_{+1}, b_{+1}).$$

Note that the buyer's cash constraint in the goods market binds; that is,  $d(\hat{m}) = \hat{m}$ . From (2),  $T = M - M_{-1} + \rho B - B_{-1}$ , and the budget constraint in the goods market satisfies  $\hat{m}\phi = z(\hat{q})$  and  $m\phi = z(q)$ . Furthermore, all agents exit the period with the same amount of money and bonds; hence  $m_{+1} = M$  and  $b_{+1} = B$ . Using these equalities, we can rewrite (A.30) as follows:

$$V_1^b(m, b) = \delta[u(\hat{q}) - z(\hat{q})] + U(x^*) - x^* \\ - \left(\frac{1}{\phi} - 1\right)[z(\hat{q}) - z(q)] + \beta V_1(m_{+1}, b_{+1}),$$

where we have used  $b = B_{-1}$  and  $m = M_{-1}$ . Another way to write this is

$$(A.31) \quad V_1^b(m, b) = \delta[u(\hat{q}) - z(\hat{q})] + U(x^*) - x^* - i[z(\hat{q}) - z(q)] + \beta V_1(m_{+1}, b_{+1}).$$

The active buyer's period surplus is  $\delta[u(\hat{q}) - z(\hat{q})]$ , but he has to pay interest  $i = \frac{1}{\phi} - 1$  on the difference  $z(\hat{q}) - z(q)$ .

Along the same lines, for a passive agent one can show that

$$(A.32) \quad V_2^b(m, b) = \delta[u(q) - z(q)] + U(x^*) - x^* + \beta V_1(m_{+1}, b_{+1}).$$

The difference between (A.31) and (A.32) is

$$(A.33) \quad V_1^b(m, b) - V_2^b(m, b) = \delta[u(\hat{q}) - z(\hat{q})] - \delta[u(q) - z(q)] - i[z(\hat{q}) - z(q)].$$

We now need to study (A.33) for the different types of equilibria. For the type I equilibrium,  $\phi$  comes from (23); thus

$$V_1^b(m, b) - V_2^b(m, b) = \Psi_1 \equiv \delta[u(\hat{q}) - z(\hat{q})] - \delta[u(q) - z(q)] > 0,$$

which is clearly strictly positive, since  $q < \hat{q}$ . For the type II equilibrium,  $\phi$  comes from (27); thus

$$(A.34) \quad \Psi_2 \equiv \delta[u(\hat{q}) - z(\hat{q})] - \delta[u(q) - z(q)] - \delta \left[ \frac{u'(\hat{q})}{z'(\hat{q})} - 1 \right] [z(\hat{q}) - z(q)].$$

For the type III equilibrium,  $\phi$  comes from (31); thus

$$\Psi_3 \equiv \delta[u(\hat{q}) - z(\hat{q})] - \delta[u(q) - z(q)] - \left( \frac{B_0}{M_0} \frac{1-n}{n} - 1 \right) [z(\hat{q}) - z(q)].$$

Note that in the type III equilibrium we have  $\delta \frac{u'(\hat{q})}{z'(\hat{q})} + 1 - \delta \geq \frac{1}{\phi} = \frac{B_0}{M_0} \frac{1-n}{n}$ . Accordingly,  $\Psi_3 \geq \Psi_2$ . Hence, it is sufficient to show that  $\Psi_2 > 0$ . To do so, rewrite (A.34) as follows:

$$u(\hat{q}) - u(q) - z(\hat{q}) + z(q) > \left[ \frac{u'(\hat{q})}{z'(\hat{q})} - 1 \right] [z(\hat{q}) - z(q)].$$

Divide both sides of the above inequality by  $\hat{q} - q$  and rearrange it to get

$$\frac{\frac{u(\hat{q}) - u(q)}{\hat{q} - q}}{\frac{z(\hat{q}) - z(q)}{\hat{q} - q}} > \frac{u'(\hat{q})}{z'(\hat{q})}.$$

The left-hand side is larger than the right-hand side, since we have assumed that  $\frac{u'(q)}{z'(q)}$  is a strictly decreasing function of  $q$ . Hence,  $\Psi_3 \geq \Psi_2 > 0$ . ■

**PROOF OF LEMMA 5.** From an active seller's decision problem in the secondary bond market,  $V_1^s(m, b) = V_2^s(\hat{m}, \hat{b})$ . In any equilibrium, the seller's budget constraint (15) holds with equality. Thus, we can use (15) to eliminate  $\hat{b}$  from  $V_2^s(\hat{m}, \hat{b})$  and get

$$(A.35) \quad V_1^s(m, b) = V_2^s\left(\phi\hat{m}, \frac{\phi m + \varphi\phi b - \phi\hat{m}}{\varphi\phi}\right).$$

Using (13), the following holds:

$$V_1^s(m, b) = \delta^s [-c(q) + \phi d] + V_3\left(\hat{m}, \frac{\phi m + \varphi\phi b - \phi\hat{m}}{\varphi\phi}\right),$$

which can be rewritten as follows:

$$\begin{aligned} V_1^s(m, b) = & \delta^s [-c(q) + \phi d] + U(x^*) - x^* + \phi\hat{m} + \frac{\phi m + \varphi\phi b - \phi\hat{m}}{\varphi} \\ & + \phi T - \phi m_{+1} - \phi\phi b_{+1} + \beta V_1(m_{+1}, b_{+1}), \end{aligned}$$

by virtue of (3) and (4).

For a passive seller, one can show that

$$\begin{aligned} V_2^s(m, b) = & \delta^s [-c(q) + \phi d] + U(x^*) - x^* + \phi m + \phi b \\ & + \phi T - \phi m_{+1} - \phi\phi b_{+1} + \beta V_1(m_{+1}, b_{+1}). \end{aligned}$$

Hence the difference  $V_1^s(m, b) - V_2^s(m, b)$  is equal to

$$V_1^s(m, b) - V_2^s(m, b) = \phi\hat{m} - \phi m + \frac{\phi m - \phi\hat{m}}{\varphi} = i(\phi m - \phi\hat{m}).$$

Note that active sellers do not carry any money into the goods market; thus  $\phi\hat{m} = 0$ . Also note that  $\phi m = z(q) > 0$ . It turns out that the above difference is positive if  $i > 0$ . ■

**B. Other Pricing Mechanisms.** Here, we discuss how the key equations change when we assume one of the other pricing mechanisms mentioned earlier. Using the Kalai bargaining solution is straightforward. Competitive pricing is a bit more involved.

**B.1. Kalai bargaining.** The Nash bargaining solution is nonmonotonic (see Aruoba et al., 2007). In contrast, the Kalai bargaining solution, also referred to as *proportional* bargaining (Kalai, 1977), is monotonic, and because of this property, it is increasingly used in monetary economics.<sup>26</sup> It can be formalized as follows:

$$(q, d) = \arg \max u(q) - \phi d$$

$$\text{s.t. } u(q) - \phi d = \theta[u(q) - c(q)] \quad \text{and} \quad d \leq m.$$

When the buyer's cash constraint is binding, the solution is  $d = m$  and

$$(A.36) \quad \phi m = z^K(q) \equiv \theta c(q) + (1 - \theta)u(q),$$

where the superscript  $K$  refers to Kalai bargaining. When the buyer's constraint (7) is binding, the Kalai bargaining solution differs from the Nash bargaining solution unless  $\theta = 0$  or  $\theta = 1$ . When the constraint is nonbinding, Nash bargaining and Kalai bargaining yield the same solution.

It is straightforward to study the model under Kalai bargaining. One only needs to replace  $z(q)$  with  $z^K(q)$  in Lemmas 1–3.

**B.2. Competitive pricing.** Competitive pricing differs from random matching and bargaining along two dimensions. Obviously, there is no random matching, meaning that agents trade with certainty, since in competitive equilibrium buyers and sellers trade against the market. To make the results comparable, however, we assume that buyers and sellers can enter the goods market only probabilistically with probability  $\delta$  and  $\delta^s$ , respectively. The benefit of this assumption is that all differences in results are due to the pricing mechanism, since the number of trades is equal under all pricing protocols.

The second difference is that there is no bargaining; instead, the competitive price adjusts to equate aggregate demand and aggregate supply. The market clearing condition for the goods market is

$$(A.37) \quad \delta(1 - n)[\pi \hat{q} + (1 - \pi)q] = \delta^s n q_s,$$

where  $\hat{q}(q)$  is the quantity consumed by a buyer who has (no) access to the secondary bond market.

Below, we show that competitive pricing yields the same allocation as random matching and bargaining if the buyers have all the bargaining power; that is,  $\theta = 1$ . In particular, the terms of trade satisfy<sup>27</sup>

$$(A.38) \quad z^C(q) \equiv q.$$

It is then straightforward to study the model under competitive pricing. One only needs to replace  $z(q)$  with  $z^C(q)$  in Lemmas 1–3.

Under competitive pricing, it is natural to interpret  $\delta$  and  $\delta^s$  as participation probabilities in the goods market. In particular, let  $\delta$  ( $\delta^s$ ) be the probability that a buyer (seller) participates in the goods market. Then the buyer's value function in the goods market is

$$(A.39) \quad V_2^b(m, b) = \delta \max_q \left[ \begin{array}{l} u(q) + V_3(m - pq, b) \\ \text{s.t. } m \geq pq. \end{array} \right] + (1 - \delta) V_3(m, b),$$

where  $p$  is the price and  $q$  the quantity of market-2 goods consumed by the buyer. The first-order condition to this problem is

$$(A.40) \quad u'(q) = p(\phi + \lambda_q),$$

<sup>26</sup> The Kalai bargaining solution is discussed in Aruoba et al. (2007) and is used, for example, in Rocheteau and Wright (2013), Lester et al. (2012), He et al. (2012), and Trejos and Wright (2012). For a textbook treatment of the Kalai bargaining solution, see Nosal and Rocheteau (2011).

<sup>27</sup> In general, the condition is  $z^C(q_s, q) \equiv c'(q_s)q$ , where  $q_s$  is a seller's production. With a linear cost function,  $c(q_s) = q_s$ , the condition reduces to  $z^C(q) \equiv q$ .

TABLE A.1  
DATA SOURCE

<i>Description</i>	<i>Identifier</i>	<i>Source</i>	<i>Period</i>	<i>Frequency</i>
AAA Moody's corporate bond	AAA	BGFRS	60:Q1-10:Q4	Quarterly
Consumer price index	CPIAUCSL	BLS	60:Q1-10:Q4	Quarterly
Government bond – three months	USGG3M	Bloomberg	60:Q1-10:Q4	Quarterly
U.S. total public debt	GFDEBTN	FMS	66:Q1–10:Q4	Quarterly
M1 money stock	MINS	BGFRS	60:Q1–10:Q4	Quarterly
Nominal GDP	GDP	BEA	60:Q1–10:Q4	Quarterly
Velocity of money	MIV	FRBSL	60:Q1–10:Q4	Quarterly

where  $\lambda_q$  is the Lagrange multiplier on  $m \geq pq$ .

The seller's value function in the goods market is

$$(A.41) \quad V_2^s(m, b) = \delta^s \max_{q_s} [-c(q_s) + V_3(m + pq_s, b)] + (1 - \delta^s) V_3(m, b).$$

The first-order condition to this problem is

$$(A.42) \quad p\phi = c'(q_s).$$

If the buyer's cash constraint is not binding, the buyer consumes the efficient quantity  $q^*$ , where  $q^*$  solves  $u'(q) = c'(q_s)$ . If the cash constraint is binding, then he spends all his money on goods purchases, and consumption is inefficiently low. Note that, in equilibrium, an active buyer will hold more money than a passive buyer. This means that  $\lambda_q > \hat{\lambda}_q$ . It then follows that  $\hat{q} > q$ .

The buyer's envelope conditions are

$$(A.43) \quad \frac{\partial V_2^b}{\partial m} = \phi \delta \frac{u'(q)}{c'(q)} + (1 - \delta) \phi \quad \text{and} \quad \frac{\partial V_2^b}{\partial b} = \phi,$$

where we have used the envelope conditions in the primary bond market and the first-order conditions in the goods market. Notice the similarity between (A.43) and (12). The two expressions are the same if  $\theta = 1$ . As a consequence of this, active buyers consume the efficient quantity in a type I equilibrium under competitive pricing, whereas they do not under bilateral matching unless  $\theta = 1$ .

The seller's envelope conditions are exactly the same as (14); that is,  $\frac{\partial V_2^s}{\partial m} = \frac{\partial V_2^s}{\partial b} = \phi$ .

Finally, by using the budget constraint of the buyer at equality  $pq = m$  and (A.42) we get

$$\phi m = z^C(q) \equiv c'(q_s) q,$$

which is equal to (A.38) for a linear cost function.

*C. Data Sources (Table A.1).* The data we use for the calibration is provided by the U.S. Department of Commerce: Bureau of Economic Analysis (BEA), the Board of Governors of the Federal Reserve System (BGFRS), the Federal Reserve Bank of St. Louis (FRBSL), the U.S. Department of the Treasury: Financial Management Service (FMS), the U.S. Department of Labor: Bureau of Labor Statistics (BLS) and Bloomberg.

As the total public debt series from the U.S. Department of the Treasury: Financial Management Service is only available from 1966:Q1, we construct the quarterly data in the period from 1960:Q1 to 1965:Q4 with the data provided by <http://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm>. The definition of quarterly data that we apply is in line with

the Federal Reserve Bank of St. Louis FRED<sup>®</sup> database and defined as the average of the monthly data.<sup>28</sup>

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<sup>28</sup> The yields of the USGG3M are annualized yields to maturity and pre-tax. The rates are comprised of Generic United States on-the-run government bill/note/bond indices with a maturity of three months.



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